OSCILLATION PERIODS OF NEUTRON STARS

The recent discovery[1-4] of celestial x-ray sources prompted various authors[5-12] to propose possible production mechanisms of these x-rays. In an earlier communication, one of us[12] suggested that some of the x-ray emission might be associated with the mechanical energy of radial oscillations of neutron stars. To investigate such a possibility, precise knowledge of the oscillation periods is important. The study of the possible effect of nuclear forces on such periods is interesting in itself. The present communication presents some results of such studies.

It is well known that general relativity is important in such condensed bodies as neutron stars. Therefore, the circular frequency for purely radial oscillations in general relativity, as given by Chandrasekhar[13-15] (the final corrected expression), was used in our calculations. Three types of nuclear forces were chosen for use in the equation of state. One, designated "Skyrme," is a three-body nuclear potential.[16] The other two are neutron-neutron potentials derived by Levinger and Simmons[17], and are designated V_β and V_γ potentials. The case of non-interacting fermions was also considered for comparison. The models with zero interactions are designated "ideal" gas models, and the others with the three types of nuclear forces are called the "Skyrme," V_β, and V_γ type models, respectively. The properties of these models are more fully described in a thesis[11] and will be published in due course.

The periods for the four kinds of models are shown as a function of the stellar gravitational mass in Figure 1; the periods are expressed in milliseconds and the masses are expressed in solar mass units. The broad horizontal portion of each curve corresponds to a series of stable neutron star models. The Skyrme-type stars have periods of 0.2 to 0.3 milliseconds and the V_γ type stars have periods of 0.4 to 0.5 milliseconds in the stable region. The typical periods of the V_β type models are about 0.3 milliseconds when the stars are massive, but for less massive models the periods are about 1 millisecond. The periods for ideal gas models vary rapidly with mass, decreasing with increasing mass to about 0.8 milliseconds. Estimates of oscillation periods that can be obtained from the classical equations (order of milliseconds)[18] are especially good for the ideal gas models. However, our present results show that we must resort to calculations of the exact general relativistic expressions to obtain more detailed quantitative information.

In a suitable equation of state the pressure is not allowed to increase without limit as the density increases, so that either the restriction $p \leq \varepsilon/3$ or $p \leq \varepsilon$ must be imposed. The periods were calculated for both restrictions on the equations of state and are shown in Figure 1. The curves denoted by (1) represent the models with the limit $p \leq \varepsilon/3$ and those by (2), with $p \leq \varepsilon$. The difference is negligible over the major portion of the stable region because these restrictions become applicable only near the massive end of the stability region for some of the equations of state used.

The square of frequency ω^2 is positive in a stable region, becomes zero at the point of instability, and is negative in the region of instability.[13-15] The period approaches infinity at the boundaries of the stable region (one or both ends of the curves in Figure 1). The curve of the Skyrme-type models with $p \leq \varepsilon/3$, however, fails to show this singularity at the massive end. Instead of going to infinity (i.e. $\omega^2 = 0$), the period approaches a finite value, as infinite central density is approached, after a number of damped oscillations. For this particular model, instability never sets in at the high density limit. All other models chosen for this investigation, however, show a singularity at the point of the major mass maximum.

The behavior on the low mass side is more complicated. In order to obtain more quantitative information in this region we must include electrons in our configuration. All present models have a pure neutron configuration. Therefore, all curves in Figure 1 are terminated near 0.2 solar masses.

In order to single out the effect of nuclear forces on the periods, the following period normalization may be used. The normalization factor, τ_n, is defined as

$$\tau_n = 2\pi/\omega_n ,$$

where

$$\omega_n^2 = AGMR^{-3}\left[3\Gamma-4-3GMc^{-2}R^{-1}\left(\frac{10}{7}\Gamma-1\right)\right] . \qquad (1)$$

The formula for ω_n^2 is the expression obtained for a homogenous fluid sphere with a constant Γ and constant energy density, if one expands the formula[5] for ω^2, subject to the condition $2GM/c^2R<1$. The third term in the expression is therefore the general relativistic effect[9], and the general relativistic effect on the periods is accounted for in this way. The factor A is a correction which accounts for the departure from homogeneity, and Γ is the ratio of specific heats.

In Figure 2, the normalized periods, τ/τ_n (with $\Gamma = \frac{5}{3}$ and $A = 1$), are plotted versus stellar mass. We note that the effects of nuclear forces are shown more clearly in this figure. Near ordinary nuclear densities the Skyrme-type potential has the largest attractive term, which decreases the pressure at a given density, the V_γ type has an attractive term of intermediate magnitude, and the V_β type has the least attractive term. One conclusion to be drawn from Figure 2 is that an attractive force tends to decrease the oscillation periods.

The calculations presented here are intended only to illustrate the importance of nuclear interaction corrections to the equation of state. It seems likely that neutron star vibration periods will be less than would be calculated for a gas of noninteracting particles. If thermal emission in the soft x-ray region should be detected from such objects, then it will become desirable to attempt to detect and measure vibration periods. With some additional indication of the mass or radius of such objects, these periods will then give information about the nuclear forces in the interiors of neutron stars.

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19 This expression is quite general and may be used for any range of mass.